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STUDY OF GODDARD RANGE AND RANGE RATE SYSTEM FOR THE ADVANCED TECHNOLOGICAL SATELLITE
(Formerly Project SYNCOM)
Second Quarterly Report
(1 January 1964-31 March 1964)

Prepared for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland
on
Contract No. NAS5-3694

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ADVANCED COMMUNICATION RESEARCH AND DEVELOPMENT

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STUDY OF GODDARD RANGE AND RANGE RATE SYSTEM FOR ATS

Second Quarterly Report

GENERAL INTRODUCTION

This report comprises ADCOM's Second Quarterly Report on investigations of the problems involved in the developement of a range and range rate tracking system for the Advanced Technological Satellite (ATS). These investigations are being carried out by members of the staff of ADCOM, Inc. under NASA Contract No. NAS 5-3694, working in close coordination with, and in direct support of, activities of members of the Network Engineering Branch, Network Engineering and Operations Division of Goddard Space Flight Center. This report presents the results of investigations carried out at ADCOM, Inc. between January 1 and March 31, 1964.

The results of work performed during this period are summarized in Chapter I and are presented in more detail in Chapters II and III. In Chapter II several proposed methods of implementing coherent doppler range rate measurements for ATS are discussed. In Chapter III the tracking performances of second-order and third-order phase-locked loop systems are compared.

Chapter I

SYNOPSIS OF RESULTS AND CONCLUSIONS

1.1 Introduction

The results achieved and the conclusions that can be drawn from the work reported here are presented briefly in this chapter. The presentation is organized into sections each of which summarizes the chapter identified by the section heading.

1.2 ATS Range Rate Measurement Techniques

Several methods of measuring range rate with the ATS system are discussed. An adaptation of the Syncom Beacon Technique is discussed, basically as a performance reference, to which the alternatives are compared. The alternatives considered are use of an extra range rate tone at 10 Mc, use of a single 10 Mc sideband in the same manner, and use of the 500 kc range tone for also providing range rate.

Range rate measurement performance is analyzed with regard to additive thermal noise errors, oscillator long and short term stability requirements, and the effects of coherent interfering signals. Major attention is given to the requirement for 0.01 m/sec resolution at synchronous ranges.

For the given system noise temperatures and received signal levels, and for the allowed measurement intervals, it is shown that additive noise does not prevent the attainment of 0.01 m/sec resolution for any of the measurement techniques being considered. Oscillator stability requirements are not as easy to define because of a lack of precise knowledge about oscillator noise mechanisms. Some reduction in oscillator noise effects can be obtained by frequency multiplication and filtering. In the one case where oscillator noise effects vary among the alternate systems, the VCO in the phase-locked loop used to extract the range rate tone should not contribute more than 0.024° rms phase noise for the 500 kc tone system or more than 0.48° rms for the 10 Mc system. C-W interfering signals falling within the PLL closed loop bandwidth should not exceed -55 db with respect to the 500 kc tone or -30 db with respect to the 10 Mc tone.

This preliminary investigation discloses no fundamental reason why a range rate resolution of 0.01 m/sec can not be obtained from doppler derived from the 500 kc tone and multiplied in frequency by a suitable factor.

1.3 Comparison of Second- and Third-Order Phase-Locked Loops

The tracking performance of second- and third-order phase-locked loops is compared, where the second-order loop has a filter of the form

$$F(s) = \frac{1 + \tau_1 s}{1 + \tau_2 s}$$

and the third-order loop has the filter form

$$F(s) = \frac{1 + \tau_1 s + (\tau_3 s)^2}{1 + \tau_2 s + (\tau_4 s)^2}$$

Phase transfer functions for a linearized model of each type of loop are given as well as effective phase noise bandwidths. A change of variables in each case allows a simplified interpretation of the error coefficients obtained from the dynamic error transfer function.

The second and third-order loops are shown to have equal errors for a constant doppler shift input if the loop gains are equal and no perfect integrators are allowed. The third-order loop is shown to have a smaller error for a constant doppler rate input. For still higher orders of input signal dynamics the error coefficients become more difficult to interpret. Expressions are provided for the first three error coefficients.

Chapter II

ATS RANGE RATE MEASUREMENT TECHNIQUES

2.1 Introduction

Several proposed methods of implementing coherent doppler range rate measurements with the ATS (Advanced Technological Satellite) R and $\dot{R}$ system will be discussed. Of particular importance are the use of an extra tone at about 10 Mc for range rate, the use of the highest ranging sidetone (500 kc) for also providing range rate, and an adaptation of the Syncom Beacon technique.

The performance of each of these methods is analyzed with regard to additive thermal noise errors, oscillator long and short term stability requirements, and the effects of coherent interfering signals. Major attention will be given to the requirement for 0.01 m/sec resolution at synchronous ranges.

2.2 Range Rate Measurement Errors

It is assumed that the range rate measuring system for ATS will measure the doppler frequency shift on a suitable frequency component by measuring the period of a fixed number of cycles of doppler plus a bias frequency. Such a measurement system is illustrated operationally in Fig. 2.1a, where the received frequency $f_o + f_D$ represents a modulation frequency or a combination of down-link carrier and beacon frequencies, as will be explained later.

Operation of the system can be explained with the aid of the waveforms in Fig. 2.1b. Let f_D be the two-way doppler shift on a transmitted frequency f_o, and let f_B be the bias frequency used in doppler measurement. The first counter generates a pair of pulses (start and stop) which are separated in time by the period of N_1 cycles of doppler plus bias. This time interval, T, is given by

$$T = \frac{N_1}{f_B + f_D} \tag{2.1}$$

The second counter will then measure the length of this time interval in units of $1/f_R$ seconds by counting the number of cycles of reference frequency f_R occurring between the start and stop pulses. Because of pulse jitter and circuit delays the time interval being measured can be in error by an amount designated as δ, and will then be denoted by

$$T' = T + \delta \tag{2.2}$$

The second counter will indicate

$$N_2 = T'f_R - Q \tag{2.3}$$

where Q is a quantization error incurred in making the count come out even. (See Fig. 2.1b)

The frequencies f_o, f_R, and f_B represent the actual frequency values at the time of the measurement, and may be slightly different from their intended values, the difference arising from oscillator drifts.

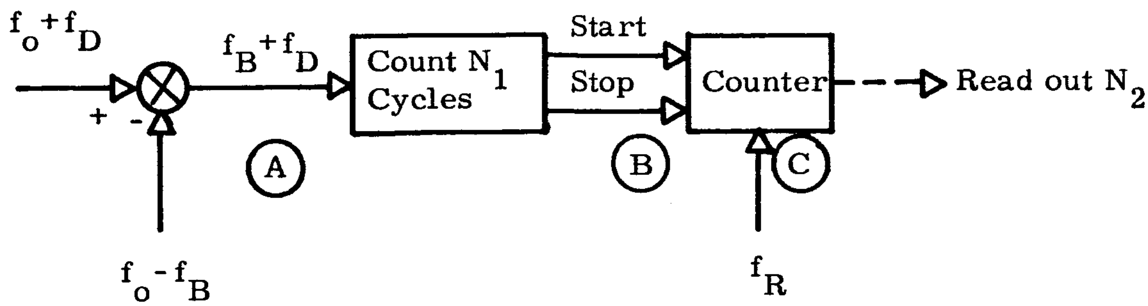


Fig. 2.1a Doppler Measurement technique.

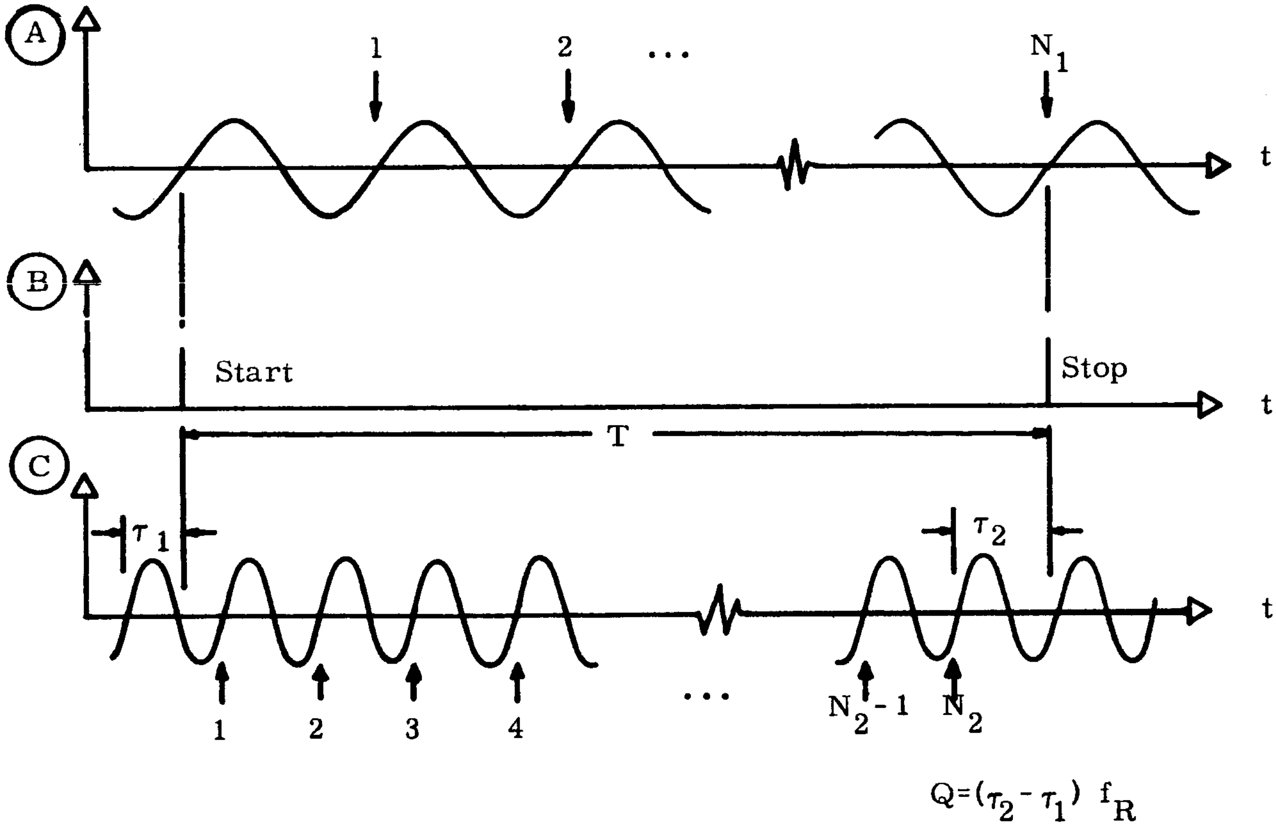


Fig. 2.1b Doppler measurement waveforms.

In calculating range rate from the counter output, N_2, the nominal or ideal frequency values will be used. These values will be designated $\hat{f}_o$, $\hat{f}_R$, and $\hat{f}_B$ to distinguish actual and ideal frequencies and allow an evaluation of the effects of frequency drifts.

We interpret the output N_2 as

$$N_2 = T \hat{f}_R = \frac{N_1 \hat{f}_R}{\hat{f}_B + f_{D_m}}$$

from which we get an indicated or measured doppler of

$$f_{D_m} = \frac{N_1 \hat{f}_R}{N_2} - \hat{f}_B \tag{2.4}$$

The value of N_2 can be found from Eqs. (2.1), (2.2), and (2.3) as

$$N_2 = \left[\frac{N_1}{f_B + f_D} + \delta \right] f_R - Q \tag{2.5}$$

Substituting Eq. (2.5) in Eq. (2.4) gives measured doppler in terms of the actual doppler and the various frequencies involved in the measurement operation,

$$f_{D_m} = \frac{N_1 \hat{f}_R}{\left[\frac{N_1}{f_B + f_D} + \delta \right] f_R - Q} - \hat{f}_B \tag{2.6}$$

requirement for a 3.6° locking error to meet range resolution requirements yields

$$(S/N)_t > 21 \text{ db} \tag{2.38}$$

Thus, the range rate resolution bound predominates over the range resolution bound. The results may be summarized in terms of signal-to-noise densities as

$$\frac{S_t}{\Phi} > 114.8 - 20 \log N_1 \quad \text{db} \tag{2.39}$$

Finally, it is noted that the analysis can be similarly extended to the 100 kc range tone. A summary of the results for this case are included in the next section.

2.3.4 Summary of Results for Additive Noise

It is now of interest to evaluate the previous techniques by establishing required signal levels for typical noise conditions. Two different noise densities are considered:

a. Φ = -181.6 dbm/cps, corresponding to a system noise temperature of 50°K assumed for the 85' antennas and low noise receiving system.

b. Φ = -179.1 dbm/cps, corresponding to a system noise temperature of 90°K assumed for the 40' antennas and receiving system.

A summary of the design evaluation for each technique is presented in Table 2.1. The signal level requirement is determined

Eq. (2.6) is not a convenient expression for evaluating the sensitivity of the measurement to parameter variations. It is desirable to achieve a form such as

$$f_{D_m} = u(f_B, f_R, \delta, Q)\, f_D + v(f_B, f_R, \delta, Q)$$

which form can be approximated in the following way. If f_B and f_R are derived from a common source such as the system master oscillator then

$$\frac{\hat{f}_R}{f_R} = \frac{\hat{f}_B}{f_B}$$

and Eq. (2.6) becomes

$$f_{D_m} = \frac{N_1}{\dfrac{N_1}{\hat{f}_B\left(1 + \dfrac{f_D}{f_B}\right)} + \delta \dfrac{f_B}{\hat{f}_B} - \dfrac{Q}{\hat{f}_R}} - \hat{f}_B \tag{2.7}$$

Since δ is small we can use the approximation

$$\delta \frac{f_B}{\hat{f}_B} \approx \delta$$

in Eq. (2.7) yielding

$$f_{D_m} \approx \hat{f}_B \left\{ \frac{1}{\dfrac{1}{1 + \dfrac{f_D}{f_B}} + \dfrac{f_B}{N_1}\left(\delta - \dfrac{Q}{\hat{f}_R}\right)} - 1 \right\} \tag{2.8}$$

If we let $x = f_D/f_B$ and $y = \hat{f}_B/N_1\ (\delta - Q/\hat{f}_R)$ the expression in brackets in Eq. (2.8) is

$$\{\cdot\} = \frac{1}{\frac{1}{1+x} + y} - 1 = \frac{x - y\,(1+x)}{1 + y\,(1+x)}$$

The term y is sufficiently small that the denominator in the last expression can be expanded in a series to yield

$$\{\cdot\} = [x - y\,(1+x)]\,[\,1 - y\,(1+x) + \ldots\,]$$
$$\approx [x - y\,(1+x)^2\,]$$

where the last step was a discarding of all terms containing y^2 or higher powers of y. Then

$$f_{D_m} \approx \frac{\hat{f}_B}{f_B}\, f_D - (\delta - \frac{Q}{\hat{f}_R})\, \frac{\hat{f}_B^2}{N_1}\,(1 + \frac{f_D}{f_B})^2 \qquad (2.9)$$

This expression will be used to evaluate the effects of oscillator drifts and instabilities on measurement performance.

2.3 Range Rate Error Due to Additive Noise

It is useful to first determine that part of the range rate error due to additive thermal noise. In a coherent doppler measurement system that determines range rate by counting doppler cycles, the rms error may be expressed in terms of the transmitted signal frequency f_o, the counting interval T and the autocorrelation function R(T) of the equivalent additive phase noise [1]

$$\dot{r}_n = \frac{c}{\sqrt{2}\,\omega_o T}\left[R(0) - R(T)\right]^{1/2} \tag{2.10}$$

where c is the velocity of light. If the signal has been extracted from the spectrum by phase-locked techniques, then R(T) may in turn be expressed in terms of the signal-to-noise density ratio S/Φ (cps) and the loop parameters as

$$R(T) = \frac{\omega_n \Phi_{if}}{8S}\, e^{-\zeta\omega_n T}\left[\frac{1+\mu^2}{\zeta}\cos\left(\sqrt{1-\zeta^2}\;\omega_n T\right) + \frac{1-\mu^2}{\sqrt{1-\zeta^2}}\sin\left(\sqrt{1-\zeta^2}\;\omega_n T\right)\right] \tag{2.11}$$

where $\mu = 2\zeta - \omega_n/K$.

In particular for a second-order loop having a damping factor of either $\zeta = 1/2$ or $1/\sqrt{2}$ and a resonant frequency ω_n much smaller than the open-loop gain K(1/sec), the rms range rate noise error is

$$\dot{r}_n \approx \frac{c}{4\pi f_o T}\left[\frac{\Phi B_n}{S}\right]^{1/2} = \frac{c}{4\pi f_o T}\left[\left(\frac{N}{S}\right)_L\right]^{1/2} \tag{2.12a}$$

for

$$T \gg \frac{4.6}{B_n} \tag{2.12b}$$

where B_n(cps) $\approx \omega_n$ (rad/sec) is the equivalent closed-loop phase noise bandwidth and $(N/S)_L$ is the noise-to-signal ratio in B_n. The requirement of Eq. (2.12b) insures that R(T)<<R(0), i.e., insures a low correlation between the noises at each end of the counting interval.

Now if the doppler measurement is done as described in Sec. 2.2, the time interval T of Eq. (2.1) can be substituted into Eq. (2.12) to yield

$$\dot{r}_n \approx \frac{c(f_B + f_D)}{4\pi f_o N_1} \left[\left(\frac{N}{S}\right)_L\right]^{1/2} \tag{2.13a}$$

for

$$N_1 >> \frac{4.6\,(f_B + f_D)}{B_n} \tag{2.13b}$$

The requirement of Eq. (2.12b) allows R(T) to be neglected resulting in considerable analytical simplification. However, it is not clear that this choice leads to a minimum $\dot{r}_n$. Clearly $\dot{r}_n$ can be reduced by requiring a high degree of noise correlation over the counting interval such that R(T) $\approx$R(0). One requirement for this case is that $\omega_n T$ be small. This effect can best be illustrated by considering the expressions resulting from several choices of ζ. The simplest expressions arise when ζ = 1 or 1/2.

When ζ = 1 use of Eqs. (2.10) and (2.11) gives

$$\dot{r}_n = \frac{\sqrt{5}\ c}{8\pi f_o T} \left[1 - e^{-\omega_n T}(1-0.6\,\omega_n T)\right]^{1/2} \left[\left(\frac{N}{S}\right)_L\right]^{1/2} \tag{2.14}$$

and when $\zeta = 1/2$ we get

$$\dot{r}_n = \frac{c}{4\pi f_o T}\left[1-e^{-\frac{\omega_n T}{2}}\cos\left(\sqrt{\frac{3}{4}}\,\omega_n T\right)\right]^{1/2}\left[\left(\frac{N}{S}\right)_L\right]^{1/2} \qquad (2.15)$$

The range rate error given by both of these expressions will go to zero as $\omega_n T$ goes to zero and similar behavior can be expected for other values of ζ. Unfortunately this effect cannot be utilized because the high correlation of noise through the measurement period, which $\omega_n T \to 0$ implies, will cause a high correlation between successive measurements which is undesirable. One additional method of reducing $\dot{r}_n$ can be found by reducing ζ below 1/2 which changes the algebraic sign of the last term in Eq. (2.11) and provides additional cancellation. However, such values of ζ are not too desirable for stable PLL operation and all considerations indicate that noise decorrelation should be emphasized as required in the approach of Eqs. (2.12). The following analysis will proceed under this restriction.

2.3.1 Analysis of the Syncom Beacon Technique

The Syncom Beacon Technique extracts the two-way doppler corresponding to the up-link carrier frequency by combining the signals from the receiver carrier and beacon loops. If the subscripts "c" and "b" are respectively used to refer to these loops, then the equivalent $(N/S)_L$ to be used in Eqs. (2.12a) and (2.13a) is found from

Fig. 2.2 to be

$$\left(\frac{N}{S}\right)_L = \left(\frac{N}{S}\right)_c + \frac{1}{4}\left(\frac{N}{S}\right)_b = \frac{\Phi B_c}{S_c} + \frac{1}{4}\,\frac{\Phi B_b}{S_b} \tag{2.16}$$

The noise bandwidths B_c and B_b must be wide enough to permit the carrier and beacon loops in the receiver to lock to the phase dynamics of their input signals. The input doppler conditions to these loops are

carrier: $2D_{6.3Gc} - 128\, D_{16.6\,Mc} = 1.66\, D_{6.3Gc}$

beacon: $253\, D_{16.6\,Mc} = 0.675\, D_{6.3Gc}$

where D represents one-way doppler. If the maximum one-way doppler shift and doppler rate are assumed to be $D = 125$ kc and $\dot{D} = 1$ kc/sec referred to 6.3 Gc, then the corresponding maximum input doppler conditions to the loops are

carrier: $f_c = 210$ kc, $\dot{f}_c = 1660$ cps/sec

beacon: $f_b = 84.5$ kc, $\dot{f}_b = 675$ cps/sec

For the type of loop under consideration, the dynamic error response to an input characterized by a doppler shift f and a doppler rate $\dot{f}$ is

$$\phi_{e,d} = \frac{360}{B_n^2}\left(\frac{f}{\tau_2} + \dot{f}\right) \text{deg} \tag{2.17}$$

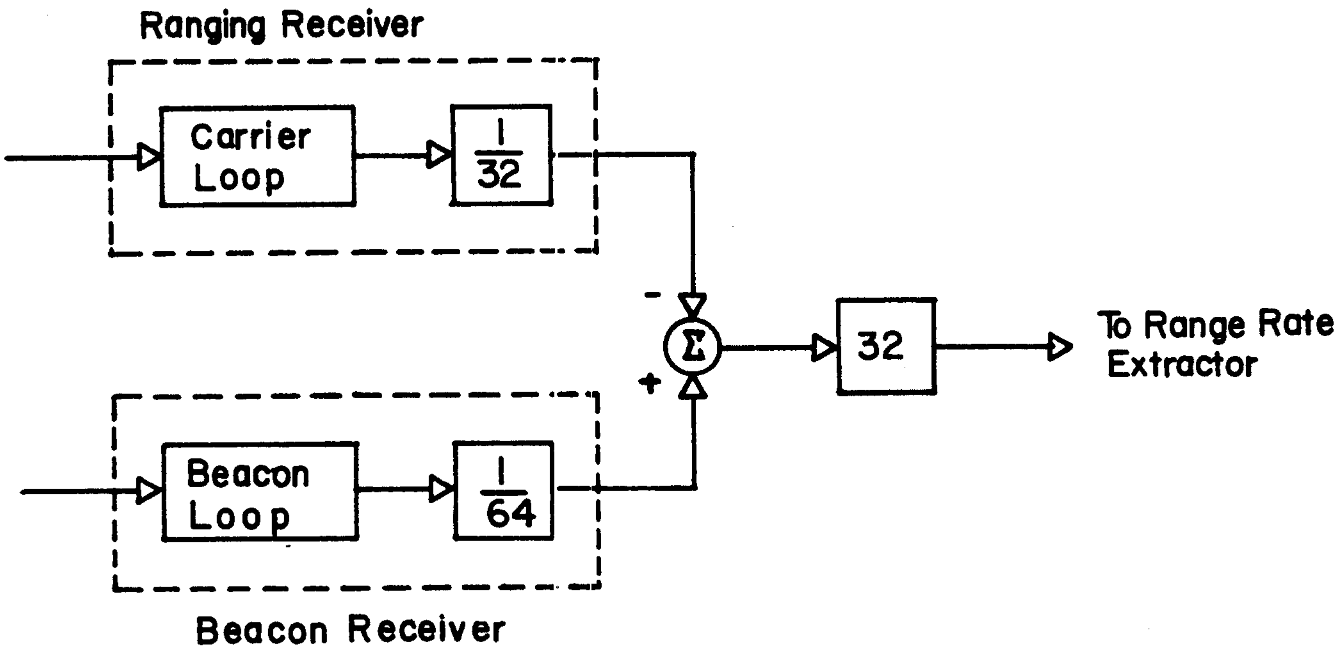


Fig. 2.2 Equivalent phase transfer model of the doppler extraction.

where $\tau_2 = K/B_n^2$ is the lag time constant of the loop filter. It is desirable to have τ_2 as large as possible, a value of 500 seconds being assumed here. If the maximum dynamic error is limited to 10^o to maintain a solid lock, and the maximum doppler shift and doppler rate occur simultaneously, then the minimum loop noise bandwidth requirements are

$$B_c \geq 273 \text{ cps} \tag{2.18a}$$

and

$$B_b \geq 174 \text{ cps} \tag{2.18b}$$

The minimum value will be used in each case to obtain maximum noise rejection, desirable both from resolution and locking considerations. These bandwidths then determine the minimum counting interval in accordance with Eqs. (2.1) and (2.13b). For Eq. (2.12a) to be valid the minimum number of cycles to be counted must satisfy

$$N_1 >> 3.31 \times 10^4 \text{ cycles} \tag{2.19}$$

The selected bias frequency is f_B = 1 Mc so that for a minimum counting interval, say $N_1 = 3.31 \times 10^5$, the corresponding counting times are T = 0.265 seconds minimum (positive doppler) and T = 0.442 seconds maximum (negative doppler).

The S/N ratio requirement is obtained from Eq. (2.13a). If the desired range rate resolution is 0.01 m/sec, and using f_o = 6.3 Gc two-way f_D = 250 kc, and the previous bandwidth choice the minimum allowable S/N ratio is

$$(S/N)_L > 112.3 - 20 \log N_1 \quad \text{db} \tag{2.20}$$

and the application of Eq. (2.16) yields

$$\frac{S_b}{\Phi} > 128.6 - 20 \log \frac{N_1}{\left(1 + 6.28 \frac{S_b}{S_c}\right)^{1/2}} \quad \text{db} \tag{2.21}$$

Under high S/N ratio conditions if $S_c \gg S_b + 8$ db

$$\frac{S_b}{\Phi} > 128.6 - 20 \log N_1 \quad \text{db} \tag{2.22}$$

Otherwise Eq. (2.21) should be used.

These S/N ratio requirements will guarantee the required range rate resolution. However, the noise locking error in the carrier and beacon loops has yet to be checked. This error is given by

$$\phi_{e,n} = 40.5 \left[\left(\frac{\Phi B_n}{S}\right)\right]^{1/2} \quad \text{deg rms} \tag{2.23}$$

If this error is limited to 10° rms, and the previous bandwidth choice is used, then the beacon and carrier S/N requirements are

$$\frac{S_b}{\Phi} > 34.5 \text{ db} \tag{2.24a}$$

and

$$\frac{S_c}{\Phi} > 36.5 \text{ db} \tag{2.24b}$$

A comparison of Eqs. (2.19) and (2.21) with (2.24a) and (2.24b) shows that this locking S/N requirement predominates over the range rate resolution requirement.

It should finally be noted that the bandwidth values used in Eqs. (2.16), (2.17) and (2.23) were assumed similar, as is the case with fixed loop filters and a strong coherent AGC maintaining constant loop gain. If a weaker AGC (e.g., limiter) or bandwidth-switching is allowed, then the bandwidths may be chosen to satisfy Eqs. (2.18a) and (2.18b) at maximum doppler but allowed to become narrower at minimum signal, provided these two conditions do not occur simultaneously. The effect is to relax the requirements of Eqs. (2.20)-(2.24b) at the expense of tightening Eq. (2.19), i.e., the power requirements are relaxed at the expense of counting time, which, however, should not be increased beyond 10 seconds. It should be recalled that the requirement of Eq. (2.19) is a direct consequence of the approximations involved in Eq. (2.12b).

2.3.2 Analysis of the 10 Mc Tone Technique

A similar approach may be used to evaluate a technique where the doppler measurement is made on a 10 Mc modulation tone.

Two cases are of interest: a DSB system (an i-f bandwidth of the order of 20 Mc) where the range rate information is only corrupted by the baseband noise around the tone, and an SSB-type system (an i-f bandwidth of the order of 10 Mc) where the carrier baseband noise directly limits the range rate resolution.

2.3.2.1 DSB System

A 10 Mc range rate tone could be included in the range tone baseband and transmitted as first order PM sidebands. The noise performance of this system is essentially characterized by

$$(N/S)_L = (N/S)_t = \frac{\Phi B_t}{S_t} \tag{2.25}$$

where S_t is referred to the receiver input and equals the total power in the two tone sidebands. While the comparison of range rate resolution and PLL locking bounds may now be obtained in terms of loop S/N ratios, the minimum loop bandwidth must still be established in order to determine the allowable counting time, i.e., the minimum N_1.

The extremes of doppler existing at the tone loop input are $2D_{10\ Mc} = 3.18 \times 10^{-3}\ D_{6.3\ Gc}$ giving a maximum tone doppler of $f_t = 397$ cps and maximum doppler rate of $\dot{f}_t = 3.18$ cps/sec. For the type of loop under consideration, a 10^o dynamic error locking bound requires

$$B_t \geq 12 \text{ cps} \tag{2.26}$$

Under the minimum bandwidth case the number of cycles to be counted must satisfy

$$N_1 >> 535 \text{ cycles} \qquad (2.27)$$

where a bias frequency of f_B = 1 kc seems a convenient choice for the existing doppler. For a minimum counting interval, N_1 = 5350, the corresponding counting times are T = 3.83 seconds minimum (positive doppler) and T = 8.85 seconds maximum (negative doppler).

The S/N ratio requirement for a 0.01 m/sec resolution is

$$(S/N)_t > 107.5 - 20 \log N_1 \quad \text{db} \qquad (2.28)$$

which, for a minimum counting interval, is $(S/N)_t > 33.0$ db. Also, the S/N ratio requirement for a 10^o rms noise locking error is

$$(S/N)_t > 12.1 \text{ db} \qquad (2.29)$$

The range rate resolution requirement of Eq. (2.28) is the more stringent. In terms of signal-to-noise density

$$\frac{S_t}{\Phi} > 118.3 - 20 \log N_1 \quad \text{db} \qquad (2.30)$$

for $8.5 \times 10^4 > N_1 > 5.35 \times 10^3$.

2.3.2.2 SSB System

The 10 Mc range rate time could also be provided separately and linearly added to the r-f signal at 10 Mc from the carrier. The noise performance of this system is characterized by

$$(N/S)_L = (N/S)_t + \frac{B_t}{B_c}(N/S)_c = \frac{\Phi B_t}{S_t} + \frac{\Phi B_t}{S_c} \qquad (2.31)$$

where $B_c >> B_t$ is assumed, the power in the tone is S_t, and the receiver uses the carrier to demodulate the 10 Mc tone which is then filtered in a 10 Mc PLL. The noise error from the carrier loop will appear as phase noise accompanying the range rate tone signal phase at the input to the tone loop. The second term in Eq. (2.31) represents the amount of this noise reproduced by the tone loop at its VCO output. Note that since only a small portion of this phase noise input is reproduced, most of the carrier noise error will appear as tone locking error.

The required loop bandwidths, established from dynamic locking error considerations, are given in Eqs. (2.18a) and (2.26), (assuming the down-link carrier is synthesized in a similar way), and the minimum number of cycles to be counted must again satisfy Eq. (2.27).

The S/N ratio requirement for a 0.01 m/sec resolution, given the previous bandwidth choice, is

$$(S/N)_L > 107.5 - 20 \log N_1 \quad \text{db} \qquad (2.32)$$

where again $f_B = 1$ kc is assumed. Application of Eq. (2.31) yields

$$\frac{S_t}{\Phi} > 118.3 - 20 \log N_1 + 10 \log \left(1 + \frac{S_t}{S_c}\right) \quad \text{db} \qquad (2.33)$$

Consider now the locking requirements. Since the tone noise locking error is equal to the tone baseband noise error, defined by Φ/S_t in B_t, plus (in a mean-square sense), the carrier noise locking error, defined by Φ/S_c in B_c, then a 10^o rms locking bound requires

$$(40.5)^2 \left(\frac{\Phi B_t}{S_t} + \frac{\Phi B_c}{S_c}\right) < (10)^2 \qquad (2.34)$$

which for the bandwidths under consideration reduces to

$$\frac{S_t}{\Phi} > 22.9 + 10 \log \left(1 + \frac{22.8\, S_t}{S_c}\right) \qquad (2.35)$$

Note that the locking bound of Eq. (2.35) approximates that of the DSB system for $S_c/S_t >> 13.7$ db.

A comparison of Eqs. (2.33) and (2.35) shows that the resolution bound predominates for

$$8.5 \times 10^4 \left(\frac{1 + \frac{S_t}{S_c}}{1 + 22.8\,\frac{S_t}{S_c}}\right)^{1/2} > N_1 > 5.35 \times 10^3$$

For $S_c/S_t >> 13.7$ db the DSB system results of Eq. (2.30) are approximated. It should be noted that the preceding results assume constant bandwidth operation.

2.3.3 Analysis of the Range Tone Technique

The signal-to-noise requirements found for the 10 Mc DSB technique are sufficiently low to suggest the possibility of using the

500 kc range tone as the range rate signal without increasing the counting time.

The doppler conditions for this tone are $2D_{500\ kc} = \frac{1}{20}(2D_{10\ Mc}) = 1.59 \times 10^{-4} D_{6.3\ Gc}$ so that $f_t = 20$ cps and $\dot{f}_t = 0.16$ cps/sec. For the type of loop under consideration the bound of a 10° dynamic locking error yields

$$B_t \geq 2.7 \text{ cps} \tag{2.36}$$

The use of this noise bandwidth with counting times less than 10 seconds will violate the assumption of Eq. (2.12b). It can be shown, however, that for the ranges of $B_n T$ products of interest Eq. (2.12a) can still be used and the results will be within 0.2 db of the results from the exact expression of Eq. (2.15) for the case $\zeta = \frac{1}{2}$. Similar agreement is to be expected for other values of damping. It is convenient to choose a bias frequency of $f_B = 50$ cps. Then the counting times of 4.29 seconds minimum and 10 seconds maximum occur with $N_1 = 300$ cycles.

The S/N ratio requirement for a 0.01 m/sec resolution is.

$$(S/N)_t > 110.5 - 20 \log N_1 \quad \text{db} \tag{2.37}$$

and for the minimum counting interval, $(S/N)_t > 60.9$ db. The requirement for a 10 second counting time is $(S/N)_t > 53.6$ db. The S/N ratio by the locking bound in the carrier/beacon technique and from the range rate resolution bound in the tone techniques. The 10 Mc tone is assumed to be at least 10 db below the carrier in the 10 Mc SSB technique.

The results shown in Table 2.1 are quite optimistic for the expected signal levels except for the 100 kc tone case, where larger counting times would be desirable. With this exception, it can be concluded that the power requirements established by additive thermal noise effects are not the crucial point in the selection of any of the techniques discussed.

It should also be emphasized that the "minimum" bandwidths used in the design may be allowed to become smaller, without necessarily increasing the counting times, when Eq. (2.12b) is not satisfied. The actual improvement capabilities are not easy to establish in general, as was previously explained.

2.4 Effects of Oscillator Drifts and Phase Jitter

The range rate errors introduced by long and short term oscillator instabilities will now be examined with regard to the various techniques of measurement discussed in Sec. 2.3. In this discussion the following assumptions will be made. First, under maximum range (synchronous) operation the range rate resolution must be 0.01 m/sec, requiring oscillator error contributions to be at least a factor of five less. At

TABLE 2.1

COMPARISON OF RANGE RATE DESIGNS FOR ADDITIVE NOISE

	Carrier/Beacon technique	10 Mc DSB technique	10 Mc SSB* technique	500 kc (DSB) technique	100 kc (DSB) technique
Loop noise band-width	B_c = 273 cps B_b = 174 cps	B_c = 273 cps B_t = 12 cps	B_c = 273 cps B_t = 12 cps	B_c = 273 cps B_t = 2.7 cps	B_c = 273 cps B_t = 1.2 cps
Bias frequency	f_B = 1 Mc	f_B = 1 kc	f_B = 1 kc	f_B = 50 cps	f_B = 10 cps
Number of cycles counted	N_1 = 331,000	N_1 = 5350	N_1 = 5350	N_1 = 300	N_1 = 60
Counting times	T=0.265-0.442 sec.	T=3.83-8.35 sec.	T=3.83-8.85 sec.	T=4.29-10 sec.	T=4.29-10 sec.
Signal required for Φ=-181.6 dbm/cps	S_c > -145.1 dbm S_b > -147.1 dbm	S_c > -145.1 dbm S_t > -134.9 dbm	S_c > -145.1 dbm S_t > -131.9 dbm	S_c > -145.1 dbm S_t > -116.3 dbm	S_c > -145.1 dbm S_t > -107.4 dbm
Signal required for Φ=-179.1 dbm/cps	S_c > -142.6 dbm S_b > -144.6 dbm	S_c > -142.6 dbm S_t > -132.4 dbm	S_c > -142.6 dbm S_t > -129.4 dbm	S_c > -142.6 dbm S_t > -113.8 dbm	S_c > -142.6 dbm S_t > -104.9 dbm

*Note: $S_c > 10\ S_t$

this range the counting time will be allowed to approach 10 seconds. Second, during the transfer ellipse the two-way doppler will be less than 8×10^{-5} of the range rate frequency. This high value is chosen to provide an added safety factor.

2.4.1 Long Term Oscillator Stability

2.4.1.1 Range Rate Frequency Source

The long term frequency stability requirements of the oscillator which generates the frequency component from which range rate is measured are not very stringent. The effect of an error in the frequency of this signal is essentially that of an error in calibration. Assuming the worst imaginable case, that of simultaneous maximum doppler and 0.01 m/sec resolution the following analysis holds. The measured range rate is

$$\dot{r}_m = \frac{c\, f_{D_m}}{2\, \hat{f}_o} \tag{2.40}$$

An error in the transmitted frequency will result in a proportionate error in doppler which will be largest at the maximum doppler shift. The doppler error Δf_D will cause a range rate error $\Delta \dot{r}$ such that

$$\frac{\Delta f_D}{2\, \hat{f}_o} = \frac{\Delta \dot{r}}{c} \tag{2.41}$$

$$= \frac{0.01}{3 \times 10^8} = \frac{10^{-10}}{3}$$

At maximum doppler $f_D/\hat{f}_o = 8 \times 10^{-5}$ and

$$\frac{\Delta f_o}{f_o} = \frac{\Delta f_D}{f_D} = \frac{2\,\hat{f}_o}{f_D}\,\frac{\Delta \dot{r}}{c} = \frac{2}{8 \times 10^{-5}} \cdot \frac{10^{-10}}{3} \approx 8.3 \times 10^{-7}.$$

A long term frequency stability of 1×10^{-7} would certainly be adequate.

2.4.1.2 Bias and Reference Oscillator

The bias and reference oscillator influences on f_{D_m} are combined in the f_B and $\hat{f}_B$ terms of Eq. (2.9). Setting δ and Q to zero in Eq. (2.9) and substituting in Eq. (2.40) yields

$$\dot{r}_m = \frac{c\, f_D\, \hat{f}_B}{2\, \hat{f}_o f_B}$$

from which follows

$$\frac{\Delta f_B}{f_B} = \frac{2\,\hat{f}_o}{f_D}\,\frac{\Delta \dot{r}}{c} \qquad (2.42)$$

This expression is identical to that for range rate frequency stability and for the same worst possible case the required frequency stability is the same, again easily satisfied by 1×10^{-7}.

2.4.2 Short Term Oscillator Stability

The short term oscillator instability of interest is that of phase drift and jitter caused by noise created in the oscillator or in associated amplifiers. The noise mechanisms and effects which cause phase jitter have not been well determined, but some conclusions can be drawn from studies of plausible noise models[2,3].

It seems safe to conclude that two noise mechanisms are of interest. One is a noise modulation caused by the coupling of noise sidebands from internal noise sources, the coupling or correlation resulting from the parametric pumping effect of a time varying conductance. The AM and FM noises so produced may or may not be correlated, depending on the form of equivalent tank circuit impedance and on any interdependence of the oscillator amplitude and frequency characteristics. The second mechanism is that of additive noise (uncorrelated noise sidebands) of which the phase noise component is of interest here.

For present purposes the various oscillators can be assumed to be frequency modulated by white gaussian noise and phase modulated by the presence of additive narrow-band gaussian noise. The rms frequency error caused by the FM noise will vary as $\frac{1}{\sqrt{T}}$ while the error from additive noise will vary as $\frac{1}{T}$, this last effect also observable in Eq. (2.10). The rms phase error will vary as $\sqrt{T}$ when caused by FM noise and, for sufficiently large values of T, will be independent of T for additive noise.

Frequency multiplication and filtering may be used to some advantage to reduce the effects of oscillator noise. The bandwidth of the additive noise will be essentially unchanged by frequency multiplication but the bandwidth of the FM noise will increase as the square of the multiplication factor. Thus, to the extent that the oscillator noise is FM by white noise, the noise power can be reduced by multiplication and filtering.

The greater part of this previous discussion has been concerned with zero-mean value noise effects. Average frequency drifts over short time intervals such that the frequency shift increases with time would have to be treated differently.

2.4.2.1 Range Rate Frequency Source

Two short term stability requirements can be placed on the source of the range rate frequency. First is a frequency stability over the transit time of the R and $\dot{R}$ signal. If the range rate frequency shifts between the times at which the signal is transmitted and is received, this shift will appear as an error in doppler. Thus Eq. (2.41) can be used with $\Delta f_D = \Delta f_o$ to yield

$$\frac{\Delta f_o}{\hat{f}_o} = \frac{2\dot{r}}{c} \tag{2.43}$$

The required stability is 6.7×10^{-11} which would be satisfied by a short term frequency stability of 1×10^{-11} over the maximum transit time (~ 0.2 second). Since the counting time will be allowed to approach 10 seconds for this case the stability could be poorer without degrading performance because of the long averaging time compared to the transit time. It is difficult to include this effect in the requirement on stability without a knowledge of the drift characteristics of the oscillator but a one-second short term stability of 1×10^{-10} would seem a conservative requirement.

The other short term stability requirement arises from the allowable jitter in establishing the time interval T. If ΔT is the allowable error in T it can be shown that

$$\Delta T = 2T_n \frac{\hat{f}_o}{\hat{f}_B} \cdot \frac{\Delta \dot{r}}{c} \tag{2.44}$$

where $\Delta \dot{r}$ is the range rate error, T_n is the nominal counting time and f_D is assumed to be zero.

This time error can be translated into a phase error referred to the bias frequency

$$\Delta\phi = 2\pi \hat{f}_B(\Delta T)$$
$$= 4\pi \hat{f}_o T_n \frac{\Delta \dot{r}}{c} \tag{2.45}$$

Considering use of the 500 kc tone for measuring range rate we may substitute $\hat{f}_o = 5 \times 10^5$, $T_n = 10$, $\Delta\dot{r} = 0.002$, $c = 3 \times 10^8$ which yields

$$(\Delta\phi)_{500\ kc} = 4.2 \times 10^{-4} \text{ rad.} = 0.024^o \text{ rms}$$

The corresponding phase error for the 10 Mc tone case would be

$$(\Delta\phi)_{10\ Mc} = 8.4 \times 10^{-3} \text{ rad.} = 0.48^o \text{ rms.}$$

Translation of this phase error into a short term stability specification is not easy because of the various noise and drift factors which enter into short term effects. An excessive requirement for the short term stability of $\hat{f}_o$ would be $\Delta\phi$ divided by the total phase change during T_n, which for the period T_n is

$$\frac{2\Delta\dot{r}}{c} = 1.33 \times 10^{-11}$$

This requirement would be independent of the range rate frequency. Since this requirement is certainly an order of magnitude high the previous requirement of 1×10^{-10} seems appropriate for f_o.

2.4.2.2 Bias and Reference Frequencies

Since the bias and reference frequency effects can be reduced to an equivalent bias effect a safe bound on their stability can be found from

$$\frac{\Delta\phi}{2\pi \hat{f}_B T_n} = \frac{\Delta T}{T_n}$$

If the ratio $\hat{f}_B/\hat{f}_o = 2.5 \times 10^{-4}$ then the short term stability requirement is 5×10^{-8} over the counting time and is independent of both range rate frequency and T.

2.4.2.3 Range Rate Frequency VCO

The phase noise specifications for the VCO in the PLL which extracts the range rate tone or carrier can be taken directly from Sec. 2.4.2.1. There it is seen that for the 500 kc system the phase noise as contributed by the VCO when the loop is locked should not exceed 0.024° rms and for the 10 Mc system should not exceed 0.48° rms.

2.4.3 Summary of Oscillator Effects

The various oscillator stability requirements were chosen such that range rate error contributions from each source would be held to 0.002 m/sec and where the measurement time was allowed to approach the limit of 10 seconds. Specifications of frequency and phase stability are somewhat difficult to manipulate, especially short term specifications, because of the noise mechanisms involved. It is also difficult to evaluate the possible benefits to be obtained from frequency multiplication and filtering although for FM noise some improvement should be possible. Preliminary investigation does indicate, however, that a range rate resolution of 0.01 m/sec should be possible from doppler derived from the 500 kc tone and multiplied in frequency by a suitable factor.

2.5 Interference Effects

Range rate errors can be caused by c-w interference which appears as phase or frequency modulation on any of the frequencies used in making the range rate measurement. The most likely way for such c-w interference to appear is on the received range rate tone, falling within the phase-locked loop bandwidth.

The effects of interference at the loop input can be estimated as follows. For the 500 kc tone case a phase shift of 2.1×10^{-3} radian during the measurement period will cause a range rate error of 0.01 m/sec. An interfering signal at the worst possible frequency and phase, causing a peak phase deviation of 1.05×10^{-3} radian to the resultant signal, could provide this amount of phase shift. Such an interfering signal would occur at $20 \log (1.05 \times 10^{-3}) = -59.6$ db with respect to the 500 kc tone. In the 10 Mc case such an interference would occur at -33.6 db.

These interference examples are absolute worst cases and in general the given interference levels would cause less error, it being possible that an arbitrarily large interference would contribute zero error. Because interfering signals could have some degree of frequency coherence with the range rate signal, as for example, intermodulation products, it does not seem appropriate to speak of mean square interference effects as averaged over a given distribution of interference frequencies and phases. It is better to require, for the 500 kc tone system, that interferences be down by 55 db including the filtering effect of the PLL. Such a specification would allow the possibility of a 0.018 m/sec error, but in general errors would be considerably less. It should be noted that PM intermodulation from the set of range tones will not be large enough at the 500 kc PLL to be troublesome.

Chapter III
COMPARISON OF SECOND- AND THIRD-ORDER PHASE-LOCKED LOOPS

3.1 Introduction

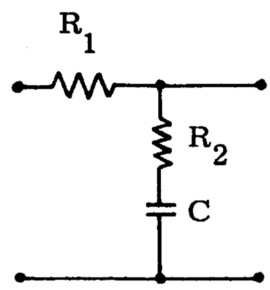
The tracking performance of a phase-locked loop (PLL) system will be investigated for two forms of the loop filter. The compromise existing between the reduction of dynamic and noise locking errors in a loop has been previously established.[4] For given input-signal phase and noise conditions, either peak or rms measures of these errors can be obtained in terms of the loop gain and time constants. The merit of a given loop filter is then determined in part by the freedom with which these errors can be manipulated. The optimum loop for a given application would be that giving the best compromise between these errors.

The conventional loop filter of Fig. 3.1 having a form

$$F(s) = \frac{1 + \tau_1 s}{1 + \tau_2 s} \tag{3.1}$$

is first considered and the loop tracking capabilities are established. A similar analysis is provided for a higher-order filter of the form

$$F(s) = \frac{1 + \tau_1 s + (\tau_3 s)^2}{1 + \tau_2 s + (\tau_4 s)^2} \tag{3.2}$$

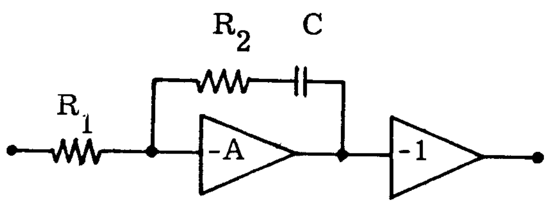


$$F(s) = \frac{1 + \tau_1 s}{1 + \tau_2 s}$$

$$\tau_1 = R_2 C$$

$$\tau_2 = (R_1 + R_2) C$$

Fig. 3.1a Passive filter for second-order PLL.



$$F(s) = A \frac{1 + \tau_1 s}{1 + \tau_2 s}$$

$$\tau_1 = R_2 C$$

$$\tau_2 = \left[(A+1) R_1 + R_2\right] C$$

Fig. 3.1b Active filter for second-order PLL.

as shown in Fig. 3.2, and a comparison between the two loop performances is presented.

3.2 The Conventional Second-Order Loop

The linear phase transfer function of the PLL[4] for the case where the loop filter has the form of Eq. (3.1) is given by

$$H(s) = \frac{1 + \tau_1 s}{1 + \left(\tau_1 + \frac{1}{K}\right)s + \frac{\tau_2}{K}s^2} \tag{3.3}$$

with K representing the open-loop gain. The effective phase noise bandwidth for this system may be found to be

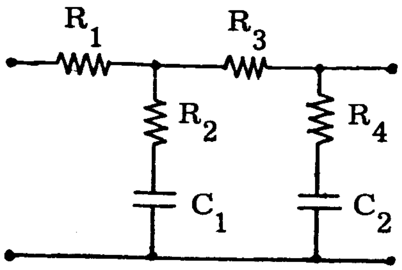
$$B_n = \frac{K}{2} \cdot \frac{1 + \frac{K\tau_1^2}{\tau_2}}{1 + K\tau_1} \quad \text{cps} \tag{3.4a}$$

$$\approx \frac{1}{2}\left(\frac{1}{\tau_1} + \frac{K\tau_1}{\tau_2}\right) \quad \text{cps for } K\tau_1 \gg 1 \tag{3.4b}$$

At this point it is useful to introduce the following change of variables into Eqs. (3.3) and (3.4a):

$$\tau_1 = \frac{1}{aB}, \quad \tau_2 = \frac{K}{a(2-a)B^2}, \quad 0 < a < 2 \tag{3.5}$$

The set of parameters (K, τ_1, τ_2) is replaced by the new set (K, B, a) which has certain advantages, as will be explained presently. The B parameter is related to the noise bandwidth B_n by



$$F(s) = \frac{1 + \tau_1 s + (\tau_3 s)^2}{1 + \tau_2 s + (\tau_4 s)^2}$$

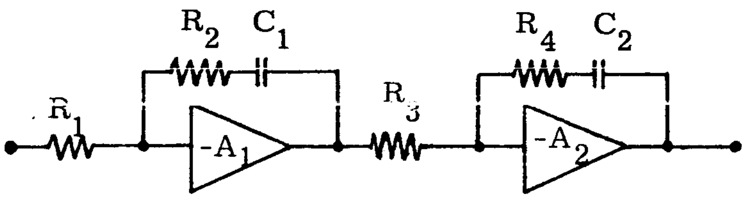
$$\tau_1 = R_2C_1 + R_4C_2$$

$$\tau_2 = (R_1 + R_2)C_1 + (R_3 + R_4)C_2$$

$$\tau_3 = \sqrt{R_2R_4C_1C_2}$$

$$\tau_4 = \sqrt{(R_1+R_2)(R_3+R_4)C_1C_2}$$

Fig. 3.2a Passive filter for third-order PLL.



$$F(s) = A_1A_2 \frac{1 + \tau_1 s + (\tau_3 s)^2}{1 + \tau_2 s + (\tau_4 s)^2}$$

$$\tau_1 = R_2C_1 + R_4C_2$$

$$\tau_2 = \left[(A_1 + 1)\, R_1 + R_2\right] C_1 + \left[(A_2 + 1)\, R_3 + R_4\right] C_2$$

$$\tau_3 = \sqrt{R_2\, R_4\, C_1\, C_2}$$

$$\tau_4 = \sqrt{\left[(A_1 + 1)\, R_1 + R_2\right]\left[(A_2 + 1)\, R_3 + R_4\right] C_1C_2}$$

Fig. 3.2b Active filter for third-order PLL.

$$B_n = \frac{B}{1 + \frac{aB}{K}} \approx B \quad \text{for } \frac{K}{B} \gg a < 2 \tag{3.6}$$

so that B may be interpreted as the noise bandwidth under the given condition. Now dynamic errors can be obtained from Eq. (3. 3) directly in terms of the noise bandwidth, and the design compromise existing between noise and dynamic errors becomes easy to illustrate and analyze. The choise of the a parameter is suggested by the fact that $B_n \approx B$. If the first term in Eq. (3. 4b) is assigned a fraction of the bandwidth, i. e., $1/\tau_1 = aB$, then the second term must be the remaining fraction, i. e., $K\tau_1/\tau_2 = (2-a)B$; and Eq. (3. 5) results. In terms of these new parameters the dynamic error transfer function is exactly given by

$$1 - H(s) = \frac{\frac{B}{K}\left(\frac{s}{B}\right) + \frac{1}{a(2-a)}\left(\frac{s}{B}\right)^2}{1 + \frac{1}{a}\left(1 + \frac{aB}{K}\right)\left(\frac{s}{B}\right) + \frac{1}{a(2-a)}\left(\frac{s}{B}\right)^2} \tag{3.7}$$

If the input signal phase $\phi_{in}(t)$ is characterized as a finite polynomial in t, then the steady state dynamic error may be conveniently expressed in terms of the derivatives of $\phi_{in}(t)$ by long division of Eq. (3. 7), as follows:

$$\phi_{e,d}(t) = \frac{1}{K}\dot{\phi}_{in}(t) + \frac{1 - (2-a)\frac{B}{K}\left(1 + \frac{aB}{K}\right)}{a(2-a)B^2}\ddot{\phi}_{in}(t) + \ldots \tag{3.8}$$

which again is an exact expression. This result may be approximated by

$$\phi_{e,d}(t) \approx \frac{1}{K}\,\dot{\phi}_{in}(t) + \frac{1}{a(2-a)B^2}\,\ddot{\phi}_{in}(t) + \ldots$$

$$\text{for } \frac{K}{B} \gg a,\ (2-a) \tag{3.9}$$

where $B \approx B_n$.

It is evident from these expressions that dynamic and noise errors can be controlled separately when the second and higher derivatives of the input phase are zero (a constant doppler shift). In this case K is chosen as large as possible for dynamic error control and B as small as possible for noise error control. The error compromise appears when higher-order derivatives occur. If the only other nonzero derivative is the second one, (a constant doppler rate) the optimum value for the a parameter is unity. If other derivatives are present, the choice of a is dictated by their relative magnitudes and the bandwidth requirements. For instance, the third term in the series expansion of Eq. (3.8) is

$$-\frac{1 - (2-a)\frac{B}{K}\left[\frac{2-3a}{2-a} + \left(\frac{aB}{K}\right)\left(2 + \frac{aB}{K}\right)\right]}{a^2(2-a)B^3}\,\dddot{\phi}_{in}(t) \tag{3.10}$$

which approximates to

$$-\frac{1}{a^2(2-a)B^3}\,\dddot{\phi}_{in}(t) \quad \text{for } \frac{K}{B} \gg 2a,\ 2-a,\ |2-3a| \tag{3.11}$$

and has a minimum value for a = 4/3.

3.3 Example of a Third-Order Loop

The linear phase transfer function of the PLL for the case where the filter form is that of Eq. (3.2) is given by

$$T(s) = \frac{1 + \tau_1 s + \tau_3^2 s^2}{1 + \left(\tau_1 + \frac{1}{K}\right)s + \left(\tau_3^2 + \frac{\tau_2}{K}\right)s^2 + \frac{\tau_4^2}{K} s^3} \tag{3.12}$$

and the effective phase noise bandwidth may be found to be

$$B_n = \frac{K}{2} \cdot \frac{1 + \frac{K\tau_3^4}{\tau_4^2} \cdot \frac{\tau_1 + \frac{1}{K}}{\tau_3^2 + \frac{\tau_2}{K}} + \frac{\tau_1^2 - 2\tau_3^2}{\tau_3^2 + \frac{\tau_2}{K}}}{1 + K\tau_1 \frac{\tau_4^2}{\tau_3^2 + \frac{\tau_2}{K}}} \tag{3.13a}$$

$$\approx \frac{K\tau_3^2}{2\tau_4^2} \cdot \frac{1 - \frac{K\tau_1\tau_3^2}{\tau_4^2} - \frac{\tau_1^2}{\tau_3^2}}{1 - \frac{K\tau_1\tau_3^2}{\tau_4^2}} \quad \text{for} \begin{cases} K\tau_1 \gg 1 \\ K\tau_3^2 \gg \tau_2 \end{cases} \tag{3.13b}$$

By analogy to the previous case, a desirable change of variable seems to be

$$\tau_1 = \frac{1}{aB}, \; \tau_3^2 = \frac{1}{bB^2}, \; \tau_4^2 = \frac{K}{cB^3} \tag{3.14a}$$

where the following relation must hold as established from Eq. (3.13b),

$$a + 2 = \frac{2ab}{c} + \frac{c}{b} + \frac{b}{a} \;;\; a, b, c > 0 \tag{3.14b}$$

The exact relation between the B parameter and the noise bandwidth is complicated and it seems sufficient to be aware that

$$B_n \approx B \text{ for } \begin{cases} \dfrac{K}{B} \gg a \\ \\ \dfrac{K}{\tau_2 B^2} \gg b \end{cases} \tag{3.15}$$

where these restrictions are a restatement, in the new variables, of the requirements of Eq. (3.13b).

In terms of the new parameters, the dynamic error transfer function and the error series expansion are given by

$$1 - H(s) = \frac{\frac{B}{K}\left(\frac{s}{B}\right) + \frac{\tau_2 B^2}{K}\left(\frac{s}{B}\right)^2 + \frac{1}{c}\left(\frac{s}{B}\right)^3}{1 + \frac{1}{a}\left(1 + \frac{aB}{K}\right)\left(\frac{s}{B}\right) + \frac{1}{b}\left(1 + \frac{b\tau_2 B^2}{K}\right)\left(\frac{s}{B}\right)^2 + \frac{1}{c}\left(\frac{s}{B}\right)^3} \tag{3.16}$$

and

$$\phi_{e,d}(t) = \frac{1}{K}\dot{\phi}_{in}(t) + \frac{1}{K}\left[\tau_2 - \frac{1}{aB}\left(1 + \frac{aB}{K}\right)\right]\ddot{\phi}_{in}(t) + \ldots \tag{3.17}$$

which again is an exact expression. This result may be approximated by

$$\phi_{e,d}(t) \approx \frac{1}{K}\,\dot{\phi}_{in}(t) + \frac{1}{K}\left(\tau_2 - \frac{1}{aB}\right)\ddot{\phi}_{in}(t) + \ldots$$

$$\text{for } \frac{K}{B} \gg a \tag{3.18}$$

where $B \approx B_n$ if $\frac{K}{\tau_2 B^2} \gg b$ also.

A comparison of Eqs. (3.9) and (3.18) shows that for equal gains the two systems will show equal errors for a constant doppler shift input. This will be the case for any order of loop filter as long as ideal integrators are not assumed. However, a smaller error coefficient may be achieved by the third-order loop for a constant doppler rate input. As an example, consider the optimum (neglecting other derivatives) choice of a = 1 in the second-order loop and arbitrarily select a = 1 for the third-order loop, which also requires 2b > c > b. The error coefficients are $\frac{1}{B^2}$ for the second-order loop and $\frac{1}{K}(\tau_2 - \frac{1}{B})$ for the third-order loop. Notice that there is an advantage in the latter case not only from the fact that $\frac{K}{B} \gg 1$ but also from the freedom in choosing τ_2, its only limitations being $\tau_2 \ll \frac{K}{bB^2}$ and physical realizability. The choice of $\tau_2 \sim \tau_1 = \frac{1}{B}$ results in a doppler rate error much smaller than with the second-order loop.

The consideration of third- and higher-order derivatives is again complicated. For instance, the third term in the series of Eq. (3.17) is

$$\frac{1}{B^3}\left\{\frac{1}{c}+\frac{B}{K}\left[\left(\frac{1}{a^2}-\frac{1}{b}\right)+\frac{2}{a}\left(\frac{B}{K}\right)+\left(\frac{B}{K}\right)^2\right]-\frac{\tau_2 B^2}{K}\left[\frac{1}{a}+2\left(\frac{B}{K}\right)\right]\right\}\dddot{\phi}_{in}(t) \tag{3.19}$$

which for the previous third-order loop example a = 1 with b << a, c << a, $\frac{c}{b} < 1.1$ reduces to

$$\frac{1}{cB^3} \quad \text{for } \frac{K}{B} \gg 2, \quad \frac{K}{\tau_2 B^2} \gg b \tag{3.20}$$

This coefficient cannot be reduced arbitrarily because c << a = 1. Note that the choice of a = 1 in this example is quite arbitrary and should not be interpreted as an "optimum" choice.

Chapter IV

PROGRAM FOR NEXT INTERVAL

1. Investigate modulation techniques for ATS which would allow "small users" of the spacecraft communication channel to operate at lower effective thresholds than is possible with the current proposed modulation. Such improvement in "small user" performance would be obtained by making it possible for them to isolate that portion of the satellite transmission in which they are interested.
2. Continue studies of the ATS range and range rate system to determine its ability to achieve the desired tracking performance through all phases of the spacecraft operation and to assure compatibility between tracking and communication portions of the system.

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